Strata: An Open-Source C++ Library for Computing Green's Functions for Layered Media

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***Abstract*—The multilayer Green's function (MGF) is often required in the integral equation modeling of structures involving layered dielectric substrates. Implementing the MGF can be a challenging and time-consuming task. We present Strata, an open-source C++ toolkit for computing the MGF in both the spectral and spatial domain. Written with modularity and readability in mind, we hope that Strata will contribute towards collaborative and community-driven advancement of research in electromagnetic solvers based on integral equations.**

I. Introduction

Electromagnetic simulation tools based on integral equations have been successful in a variety of applications, ranging from the design of antennas to the analysis of integrated circuit components. When a structure is embedded in a layered dielectric substrate, as is often the case, the substrate may be modeled with the use of the multilayer Green's function (MGF) [1]. Multilayer problems also arise in other fields such as geosensing and biological tissue modeling. However, unlike the homogeneous Green's function, the MGF cannot be expressed in closed form. Computing the MGF requires either numerical integration in the complex plane [2], or approximation techniques such as the discrete complex image method [3]. Likewise, the implementation of codes for computing the MGF is challenging and time-consuming, and often a bottleneck in the development pipeline. The significant time investment required for in-house codes may hinder experimentation with new ideas relating to the MGF, or discourage contributions from newcomers in the field. We are not aware of any open-source or freely-available MGF codes compatible with high-performance integral equation solvers.

In this work, we present a new open-source C++ framework, named Strata [4], for computing the MGF in both spectral and spatial domains. Drawing from a wealth of literature in the field, Strata is based on proven and successful algorithms related to the MGF, such as numerical integration based on partition-extrapolation [2], and the DCIM [3]. Strata is written with special attention to modularity and readability, to allow users not only to apply the code as-is, but also to modify and enhance the code with new features and improvements. We hope that Strata will contribute to the scientific community in the following ways:

- As a foundation for developing and testing new ideas and research related to the MGF.
- As a source of benchmark data to assess the accuracy and performance of novel MGF formulations and algorithms.
- As a black-box library allowing researchers in various fields to focus on their chosen subjects, without having to allocate significant resources towards the reproduction of existing codes.
- We also hope that Strata promotes the spirit of collaboration and easier access to specialized codes.

II. Description and Features

Strata encodes the commonly-used mixed-potential representation of the MGF [1]. This representation involves the dyadic MGF $\overline{\overline{\mathbf{G}}}_A$ associated with the magnetic vector potential, and the scalar MGF G_ϕ associated with the scalar electric potential. In some applications, such as surface integral formulations which involve the double-layer potential operator [5], it is also necessary to compute the dyadics $\overline{\overline{\mathbf{G}}}_{\mathrm{EM}}$ and $\overline{\overline{\mathbf{G}}}_{\mathrm{HJ}}$ [1], which are associated with the curl of $\overline{\overline{\mathbf{G}}}_A$, and are also available in Strata. It is assumed that the layered medium extends to infinity in the x and y directions, while the stratification is along the z direction. The components of $\overline{\overline{\mathbf{G}}}_A$, $\overline{\overline{\mathbf{G}}}_{\mathrm{EM}}$ and $\overline{\overline{\mathbf{G}}}_{\mathrm{HJ}}$, and also G_ϕ, can all be written as a Sommerfeld integral [6]

$$G(k, \vec{r}, \vec{r}\,') = \int_0^\infty dk_\rho \, \widetilde{G}(k_\rho, z, z') \, J_\nu(\rho k_\rho) \, k_\rho^{\nu+1}, \quad (1)$$

where $\widetilde{G}(k_\rho, z, z')$ is the spectral domain component related to $G(k, \vec{r}, \vec{r}\,')$ through a Fourier transform in the x and y directions [1]. The function $J_\nu(\rho k_\rho)$ is the Bessel function of first kind and order ν. Quantity k is the wave number associated to the source layer, k_ρ is the lateral component of the wave vector, and $\rho = \sqrt{(x-x')^2 + (y-y')^2}$. Primed and unprimed coordinates represent source and observation points, respectively. Strata computes (1) in three phases:

A. Layer Definition

The first phase involves the definition of the geometry and material properties of the layered medium. These properties are described in an input text file in the YAML[1] format, which is designed for clarity and readability. The YAML layer definition file includes the height, elevation and material properties of each layer, and well as the properties of the upper and lower half spaces surrounding the layered substrate.

[1]https://yaml.org/

B. Spectral Domain MGF Computation

The second phase involves computing $\widetilde{G}(k_\rho, z, z')$ for any or all of $\overline{\overline{\mathbf{G}}}_A$, $\overline{\overline{\mathbf{G}}}_{\mathrm{EM}}$, $\overline{\overline{\mathbf{G}}}_{\mathrm{HJ}}$, or G_ϕ. In addition to routines which directly compute $\widetilde{G}(k_\rho, z, z')$, Strata also provides access to subroutines which compute intermediate quantities, such as the reflection coefficient and transmission line impedance associated with each layer interface [6]. This allows the easy extension of Strata to incorporate additional formulations of the MGF, such as $\overline{\overline{\mathbf{G}}}_{\mathrm{EJ}}$ [1].

C. Spatial Domain MGF Computation

In the third phase, the spectral-domain MGF $\widetilde{G}(k_\rho, z, z')$ is transformed to the spatial domain, $G(k, \vec{r}, \vec{r}\,')$, through (1). The user has the option to extract the quasistatic part of the MGF in the spectral domain, and add it back in closed-form in the spatial domain [7]. Furthermore, Strata allows separate treatment of the singularities arising in the MGF. This allows users to incorporate standard singularity extraction or cancellation techniques [8] during the direct integration phase of integral equation solvers. To this end, the singular terms of the MGF can be computed as a Taylor series, with the singular term of the series extracted out. The user may then treat the singular term separately as desired. Strata provides three algorithms for computing $G(k, \vec{r}, \vec{r}\,')$ from (1):

1) Direct Integration: The algorithms described in [2] have been implemented in Strata to compute (1) with the partition-extrapolation scheme. The user is provided easy access to various control parameters, such as the tolerance to determine convergence during the numerical integration, and the shape of the path along which the integral is performed. The user has also access to the underlying complex-plane integration routines to easily extend Strata's functionality.

2) Discrete Complex Image Method (DCIM): The two-level DCIM [3], as well as a three-level extension [9], have been implemented in Strata. In the DCIM, $\widetilde{G}(k_\rho, z, z')$ is approximated as a sum of exponential terms of the form $\alpha \exp\left(-jk_z\beta\right)/k_z$, where k_z is defined via $k^2 = k_\rho^2 + k_z^2$. Terms α and β are complex fit parameters found via the generalized pencil-of-functions method (GPOF) [10]. Each exponential term is then converted to spatial domain in closed form with the Sommerfeld identities [11], leading to significant computational savings compared to direct integration. Strata provides access to intermediate subroutines for the GPOF procedure and associated settings, so that new multilevel DCIM algorithms such as the ones in [9] can be readily developed.

3) Interpolation Tables: Strata supports the precomption of interpolation tables via either direct integration or the DCIM. The interpolation tables can be optionally stored to disk and loaded later on. This can save a significant amount of computational effort when simulating different structures embedded in the same stack-up, since the interpolation tables can be reused.

III. Representative Examples

To validate accuracy, the MGF curves reported in Fig. 11 in [7] and Fig. 3 in [12] were digitized, and compared against the corresponding components of the MGF computed via Strata. The results in Fig. 1 show an excellent agreement between our implementation and the literature. Several more validation test cases from literature are included in Strata.

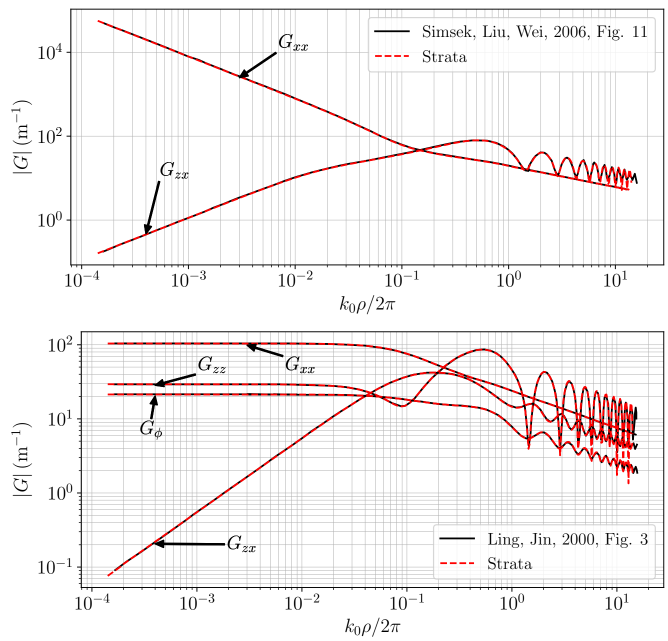

Fig. 1. Accuracy validation for Strata. Top: Comparison to Fig. 11 in [7]. Bottom: Comparison to Fig. 3 in [12].

References

[1] K. A. Michalski and J. R. Mosig, "Multilayered media Green's functions in integral equation formulations," *IEEE Trans. Antennas Propag.*, vol. 45, no. 3, pp. 508–519, Mar. 1997.

[2] ——, "Efficient computation of Sommerfeld integral tails - methods and algorithms," *J. Electromagn. Waves Appl.*, vol. 30, no. 3, pp. 281–317, 2016.

[3] M. I. Aksun, "A robust approach for the derivation of closed-form Green's functions," *IEEE Trans. Microw. Theory Tech.*, vol. 44, no. 5, pp. 651–658, May 1996.

[4] https://github.com/modelics/strata.

[5] D. Colton and R. Kress, *Integral Equation Methods in Scattering Theory*. Hoboken, NJ, USA: Wiley, 1983.

[6] K. A. Michalski and D. Zheng, "Electromagnetic scattering and radiation by surfaces of arbitrary shape in layered media. I. Theory," *IEEE Trans. Antennas Propag.*, vol. 38, no. 3, pp. 335–344, Mar. 1990.

[7] E. Simsek, Q. H. Liu, and B. Wei, "Singularity subtraction for evaluation of Green's functions for multilayer media," *IEEE Trans. Microw. Theory Tech.*, vol. 54, no. 1, pp. 216–225, Jan. 2006.

[8] W. C. Gibson, *The Method of Moments in Electromagnetics*. Boca Raton, FL, USA: CRC press, 2014.

[9] A. Alparslan, M. I. Aksun, and K. A. Michalski, "Closed-form Green's functions in planar layered media for all ranges and materials," *IEEE Trans. Microw. Theory Tech.*, vol. 58, no. 3, pp. 602–613, Mar. 2010.

[10] Y. Hua and T. K. Sarkar, "Generalized pencil-of-function method for extracting poles of an EM system from its transient response," *IEEE Trans. Antennas Propag.*, vol. 37, no. 2, pp. 229–234, Feb. 1989.

[11] M. Yuan, T. K. Sarkar, and M. Salazar-Palma, "A direct discrete complex image method from the closed-form Green's functions in multilayered media," *IEEE Trans. Microw. Theory Tech.*, vol. 54, no. 3, pp. 1025–1032, Mar. 2006.

[12] F. Ling and J.-M. Jin, "Discrete complex image method for Green's functions of general multilayer media," *IEEE Microw. Guided Wave Lett.*, vol. 10, no. 10, pp. 400–402, Oct. 2000.