Accelerated Electromagnetic Analysis of Interconnects in Layered Media using a Near-Field Series Expansion of the Green's Function

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***Abstract*—An efficient method for evaluating the multilayer Green's function is proposed, with application to interconnect modeling with integral equations. A series expansion of Bessel functions for near-field interactions provides demonstrated efficiency in a realistic example.**

***Index Terms*—interconnect modeling, surface integral equation method, multilayer Green's function**

I. Introduction

The increasing complexity of embedded on-chip interconnects demands fast and accurate numerical solvers to model their electromagnetic behavior. Although surface integral methods coupled with acceleration techniques, such as the adaptive integral method (AIM) [1] and the fast multipole method (FMM) [2] have been successful in modeling large structures, the presence of surrounding stratified media greatly increases the computational cost in general.

Analytical expressions are not available for the multilayer Green's function (MGF), which involves semi-infinite Sommerfeld-type integrals. These integrals must be evaluated for every possible combination of source and observation points on a given mesh. Direct numerical integration thus leads to large computation times. A popular and efficient technique for computing the MGF is the discrete complex image method (DCIM) [3], in which a fitting technique is used to approximate the MGF as a sum of exponential factors, called images, that correspond to spherical waves. For these factors, the Sommerfeld integral is known analytically. However, the computational cost associated with this technique is still significantly greater than the free space case, since the number of images required may be as large as 10-15 for realistic substrates used in integrated circuits.

More recently, series expansions have been proposed for approximating the Green's function more efficiently [4], [5]. In [4], the expansion is applied to the homogeneous Green's function for the partial element equivalent circuit method. The method in [5] is for layered media, but requires interpolation in addition to a series expansion, thus invoking two levels of approximation.

When integral equation methods are coupled with acceleration techniques such as AIM or FMM, reaction integrals are computed in two different ways depending on the distance between source and observation points. In the far field, an approximation is used to enable quick computation using, for example, a fast Fourier transform. In the near field, such approximation is not valid, and Sommerfeld integrals must be evaluated accurately. This computation can be very time consuming for realistic structures, and often the main bottleneck of the overall analysis.

In this work, we propose a new method to accelerate the computation of the MGF for application to on-chip interconnects. We take advantage of the fact that only near-field interactions need to be computed accurately. We apply a series expansion of Bessel functions of first kind that appear in the MGF, of orders 0 and 1. Coupled with the analytical addition of quasistatic contributions, this leads to a significant speed-up and predictable error-control as compared to DCIM, with only one level of approximation and no interpolation. The performance of this technique is demonstrated on a realistic interconnect structure, and compared against DCIM and an industry-standard finite element tool. Significant speed-up with negligible change in accuracy is demonstrated.

II. Formulation

We consider expressions for the MGF proposed in formulation C of [6], which consists of a dyadic term $\overline{\overline{\mathbf{G}}}$ and a scalar term G_ϕ. However, it should be noted that the method presented here is extensible to any valid formulation.

Assuming that the dielectric layers are stacked along the z axis, the dyadic MGF can be written as

$$\overline{\overline{\mathbf{G}}} = \begin{bmatrix} G_{xx} & 0 & G_{xz} \\ 0 & G_{yy} & G_{yz} \\ G_{zx} & G_{zy} & G_{zz} \end{bmatrix}. \tag{1}$$

Each component of $\overline{\overline{\mathbf{G}}}$, as well as G_ϕ, has the form

$$G_l\left(k, \mathbf{r}, \mathbf{r}'\right) = \int_0^\infty dk_\rho J_v\left(\rho k_\rho\right) \tilde{G}_l\left(k_\rho, z, z'\right) k_\rho^{v+1}. \tag{2}$$

The function $J_v\left(\rho k_\rho\right)$ is the Bessel function of first kind and order v, and $\tilde{G}_l\left(k_\rho, z, z'\right)$ is the spectral MGF corresponding to $G_l\left(k, \mathbf{r}, \mathbf{r}'\right)$. Subscript l refers to any of $\{xx, yy, zz, xz, yz, zx, zy, \phi\}$. Quantity k is the wave number, and k_ρ is the wave number in the lateral (xy) plane. Primed and unprimed coordinates represent source and observation points, respectively, and $\rho = \sqrt{(x-x')^2 + (y-y')^2}$. Bessel

functions of order $v = 0$ are used for diagonal components and for G_ϕ, while $v = 1$ for off-diagonal components.

Either type of Bessel function can be expanded as a Taylor series centered at $\rho k_\rho = 0$ [7],

$$J_v(\rho k_\rho) = (0.5\rho k_\rho)^v \sum_{i=0}^{\infty} \frac{\left(-0.25\rho^2 k_\rho^2\right)^i}{i!\,(v+i)!}. \tag{3}$$

This expansion allows us to write the Bessel function as a product of terms that depend only on ρ, and terms that depend only on the simulation frequency via k_ρ,

$$J_v(\rho k_\rho) = \sum_{i=0}^{\infty} \rho^{v+2i} k_\rho^{v+2i} \frac{(0.5)^v (-0.25)^i}{i!\,(v+i)!}. \tag{4}$$

With this separation of terms, we can extract the summation and ρ-dependent part out of the Sommerfeld integral after substituting (4) into (2),

$$G_l(k, \mathbf{r}, \mathbf{r}') = \sum_{i=0}^{\infty} \rho^{v+2i} \frac{(0.5)^v (-0.25)^i}{i!\,(v+i)!} \int_0^{\infty} dk_\rho k_\rho^{v+2i} \tilde{G}_l(k_\rho, z, z')\, k_\rho^{v+1}. \tag{5}$$

The integrand in (5) now only depends on the simulation frequency and coordinates along the direction of stratification. The Sommerfeld integral can thus be precomputed for a predetermined set of z–z' pairs, rather than every possible combination of source and observation points. This is particularly advantageous for on-chip structures, which are often much smaller along the direction of stratification than in the lateral directions. To precompute these semi-infinite integrals, we use the partition-extrapolation approach in [8].

It is known [9] that in the quasi-static limit, the MGF can be written analytically in the spatial domain. These quasi-static terms have a significant contribution in the near-field, and can be leveraged to further improve the approximation. The spatial domain quasi-static contribution can be expressed as a sum of terms having the form

$$G_{\mathrm{qs},i}(k, \mathbf{r}, \mathbf{r}') = \Gamma_i \frac{\mathrm{e}^{-jk\gamma_i(\rho,z,z')}}{\gamma_i(\rho, z, z')} \tag{6}$$

for diagonal components of the MGF, and

$$G_{\mathrm{qs},i}(k, \mathbf{r}, \mathbf{r}') = \frac{\Gamma_i}{\rho}\left(1 - \frac{\zeta_i(z, z')}{\gamma_i(\rho, z, z')}\right) \tag{7}$$

for off-diagonal components. Quantity Γ_i is a Fresnel reflection coefficient for the i^{th} quasi-static term, and depends on layer properties. Term ζ_i is a linear combination of z and z' based on the source and observation layers, and $\gamma_i = \sqrt{\rho^2 + \zeta_i^2}$. Detailed expressions can be found in [9]. We extract the corresponding quasi-static contributions in the spectral domain prior to precomputing the integral in (5), and add them back analytically as per (6) and (7). This causes the spectral functions $\tilde{G}_l(k_\rho, z, z')$ to decay to 0 significantly faster along the real k_ρ axis, thus speeding up the Sommerfeld integrals. Further, this precludes the need to sample $\tilde{G}_l(k_\rho, z, z')$ for large values of k_ρ. Thus, the argument of the Bessel functions, ρk_ρ, can be kept sufficiently small, to ensure that the approximation in (4) is valid even for a small number of expansion terms.

TABLE I
EXAMPLE IN SEC. III: PROPERTIES OF THE SUBSTRATE.

ε_r	μ_r	σ (S/m)	Height (μm)
11.5	1.0	0.01	3
9.8	1.0	0.001	4
12.5	1.0	0.1	10
6.0	1.0	0.0001	9
4.4	1.0	0.0	4

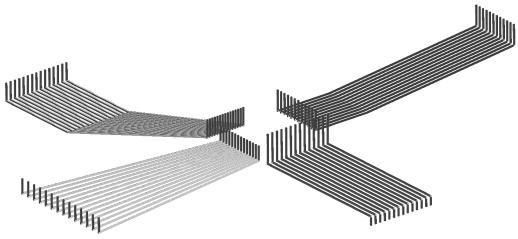

Fig. 1. Interconnect network geometry.

Compared to DCIM, the proposed method provides two significant advantages:

1) In DCIM, it is typically not known *a priori* how many images will be required to model a given layer configuration. Complicated stack-ups may require in excess of 10–15 images, and the matrix assembly cost is proportional to this number. With the proposed series expansion, however, the number of terms required is fixed and can be as small as 2–3 for on-chip applications.
2) The series expansion allows for direct error control by picking an appropriate number of expansion terms. In DCIM, errors are controlled in an indirect way during the fitting procedure, by retaining a heuristically-chosen number of singular values during the SVD step.

III. NUMERICAL RESULTS

To demonstrate the proposed methodology, we consider a network of 55 copper on-chip interconnects with a cross section size of $1\,\mu$m and conductor lengths of $150\,\mu$m on average. The geometry is shown in Fig. 1. The structure is embedded in a stack-up shown in Table I. The layers are bounded by free space above and below, and lossy dielectrics are included for generality. All tests are performed on a 3.6 GHz desktop computer with 24 GB physical memory.

The interconnect network is simulated with the surface integral approach proposed in [10], accelerated with AIM. The differential surface admittance operator proposed in [11] is utilized to model the skin effect in conductors. The proposed series approximation is used to accelerate the near-field matrix fill, while DCIM is employed for far-field interactions. The scattering (S) parameters are extracted from 800 MHz to 40 GHz. For validation, we compare these results to the case

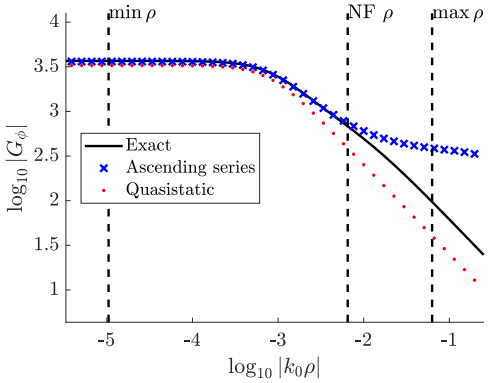

Fig. 2. Ascending series approximation for G_Φ, with quasistatic contribution.

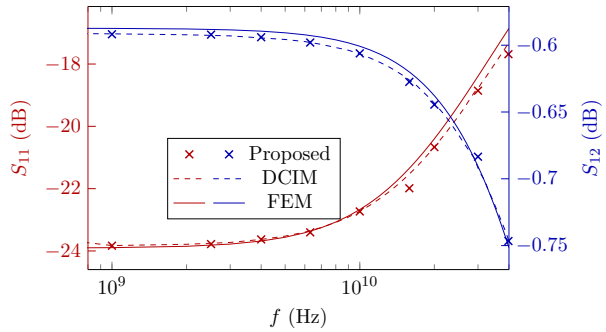

Fig. 3. Selected transmission (S_{11}) and reflection (S_{12}) parameters for a four-port on-chip interconnect, compared with HFSS.

when DCIM is used for both the near-field and far-field interactions. Additionally, we validate the results against a commercial finite element tool, Ansys HFSS.

The scalar component of the MGF, along with contributions of quasi-static terms and the series approximation, is plotted in Fig. 2. Only the first two terms were used for the expansion in (4). The vertical dashed lines indicate the smallest ($\min \rho$), largest ($\max \rho$) and near-field (NF) distances relevant to the geometry of the interconnect network. Similar or better results were obtained for other MGF components, and are omitted here for brevity. Clearly, the series expansion is sufficient for use in the near-field region.

The S parameters obtained with the proposed technique are shown in Fig. 3, and are in excellent agreement with both DCIM and HFSS. In Table II, the CPU times for matrix fill are reported for a representative frequency point (1 GHz), for the proposed method and DCIM. Also reported is the CPU time for precomputation of Sommerfeld integrals as well as DCIM. Although the precomputation time is larger for the proposed method, the matrix fill time is the bottleneck in general. Subsequently, the $2.55\times$ speed-up in matrix fill provided by the proposed method yields a significant computational advantage compared to DCIM. In the proposed method, the near-field matrix fill takes only $38\,\%$ of the total time, while with DCIM, the near-field part comprises $63\,\%$ of the total time. The proposed method is thus a significant step towards ensuring that the near-field matrix fill is no longer a bottleneck in the simulation of large, realistic structures.

TABLE II
CPU TIMES FOR PROPOSED METHOD AND DCIM, FOR $f = 1\,\text{GHz}$.

	Proposed	DCIM	Speed-up
Precomputation (min)	5.7	3.5	0.62x
Near-field interactions (min)	24.6	63.8	2.55x

IV. Conclusions

An efficient method to compute the multilayer Green's function in an integral equation context is described, based on a series expansion of the Bessel function for near-field interactions. The proposed method is suitable for on-chip applications, as demonstrated in a realistic test case. In addition to its robustness, direct error control and ease of implementation, it provides a significant advantage over the popular discrete complex image method (DCIM) in terms of CPU time.

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